Answer Key”””””

Assignment for Freshman students Total weight:15%

Date:29/11/2019

To be submitted on 10/12/2019

1. Consider the statement ‘’**there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x” as a logical expression’’***.* Then answer the following:
   a. Express the statements in terms of quantifiers

   Solution

Let y stands for any number and we are also given with x as number.

The conditions we are given with are there is x such that $x + y = y$ and $xy = x$

This statement can be shortened by using quantifiers as $(\exists x)\forall(y)(x + y = y \wedge xy = x)$

   b. Find the truth value that constructed in (a)

The truth value of $(\exists x)\forall(y)(x + y = y \wedge xy = x)$ is true because we can get at least one example that satisfies the conditions. For instance, for x=0 and y=0

At this case the universe must be considered, it holds or true if the universe of discourse is $N, \mathrm{Z}, \mathrm{W}, \mathrm{R}, \mathrm{Q}$ and it is not true if z^{+}

2. Let $\mathrm{P}(x, y)$ denote ‘’x is the factor of y’’ where $x\epsilon\{1,2,3, \ldots ..\}$ and $y \in \{2,3,4, \ldots .\}$. and let $\mathrm{Q}(y)$ denote ‘’ $\forall x(P(x, y) \Longrightarrow ((x = y) \vee (x = 1))''$. When does Q(y) is true?

Solution

It is true when y only prime.

3. In Addis Ababa city in Nov.28,2019 of 10,000 families it was found that 40% families buy newspaper A,20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find
    a. The number of families which buy newspaper A only.

Solution

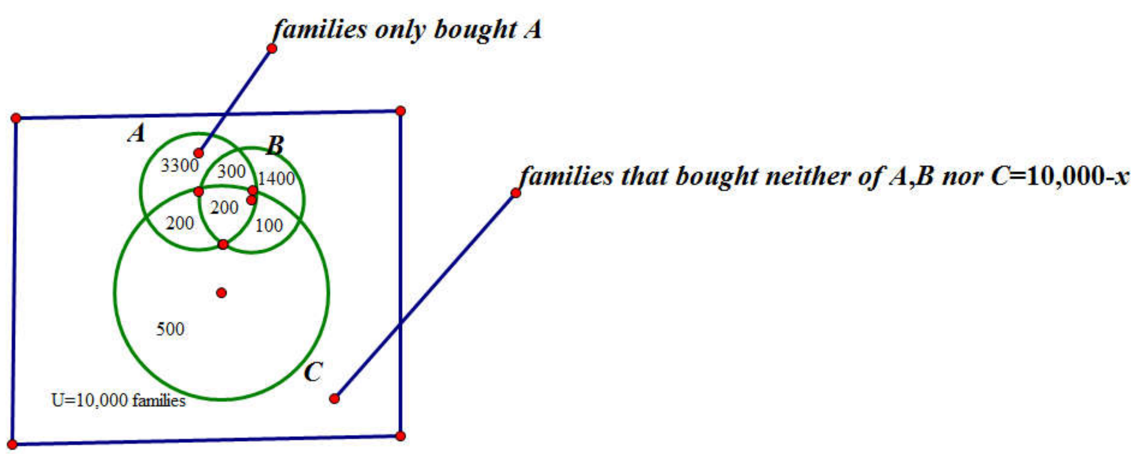
Let U=be several families buying newspapers.

$$\mathrm{U} = n(\mathrm{A} \cup \mathrm{B} \cup \mathrm{C}) + \mathrm{n}(\mathrm{A} \cap \mathrm{B}) + \mathrm{n}(\mathrm{B} \cap \mathrm{C}) + \mathrm{n}(\mathrm{A} \cap \mathrm{C}) + \mathrm{n}(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = 10{,}000$$

By definition n

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - (n(A \cap B) + (A \cap C) + (B \cap C)) + n(A \cap B \cap C)$$
$$\Rightarrow n(A) = n(A \cup B \cup C) - (n(B) + n(C) - (n(A \cap B) + (A \cap C) + (B \cap C)) + n(A \cap B \cap C))$$
$$those, who, bought, newpapers, A, are, n(A) = 4000, n(B) = 2000, n(C) = 1000$$
$$but, those, who, bought, newspaper, A\&, others, are, n(A \cap B) = 500, n(A \cap C) = 400, n(A \cap B \cap C) = 200$$

By using the above calculation or by using Ven-diagram we can illustrate the answer as following.



As we see that the number of families that bought only news paper $A = 3300$

    b. The number of families which buy none of A, B and C

By using the above Ven-diagram the number of families which buy none of A, B and C can be calculated by $10{,}000 - (3300 + 300 + 200 + 200 + 100 + 500 + 1400) = 4{,}000$

4. Two finite sets have m and n number of elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

Solution

Since, let $A \& B$ be such sets, that is $n(A) = m, n(B) = n$

$$n(P(A)) = 2^m, n(P(B)) = 2^n \Rightarrow n(P(A)) = 2^m - n(P(B)) = 2^n = 56$$

So $\Rightarrow 2^n(2^{m-n} - 1) = 2^3 \times 7$

$$\Leftrightarrow 2^n = 2^3 \vee 2^{m-n} - 1 = 7 \Rightarrow n = 3, m = 6$$

This is the required answer.

5. Let U be the set of all boys and girls in Wachemo University, G be the set of all girls in the University, B be the set of all boys in the University, and S be the set of all students in the University who want to join Mathematics Department. Some, but not all, students in the University want to join Mathematics Department. Draw a Venn diagram showing one of the possible interrelationships among sets U, G, B and S.

Solution

Let U= the set of all boys and girls in Wachemo University

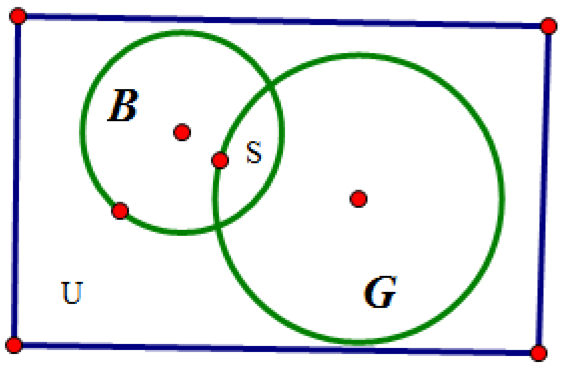
Let G = the set of all girls in the University

B = the set of all boys in the University

From this we see that $U = B \cup G$ in the given university

Let S =the set of all students in the University who want to join Mathematics Department

From this we see also that in the set S we may have both boy and girl that is $S = B \cap G$ and we have conditions that say some, but not all, students in the University want to join Mathematics Department. In general, one possible ven-diagram presentation can be figured as follows:



6. Let $z_1 = 2 + 2i \ and \ z_2 = 2 - 2i$ be the complex number. Then compute

   For Natural science extreme only

a. $z_1 * z_2$ in polar form

Solution

$z_1 * z_2 = r_1 \times r_1(\cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)$ where $\theta_1 = tan^{-1}(1) = \frac{\pi}{4}$ for$z_1 = 2 + 2i$

And$\theta_2 = tan^{-1}(-1) = \frac{-\pi}{4}$ for$z_1 = 2 - 2i$ as it finds in fourth quadrant as it is depicted in the following z-plane or complex plane. Clearly both complex has the same modulus that is $2\sqrt{2}$

Thus

$z_1 * z_2 = 8(cos0 + isin0)$ is the answer.

b. Find the fourth root of the product the two complex number plus two times $(3 - 3i)$

   Solution

   Mathematically the questions say that find the fourth root of let

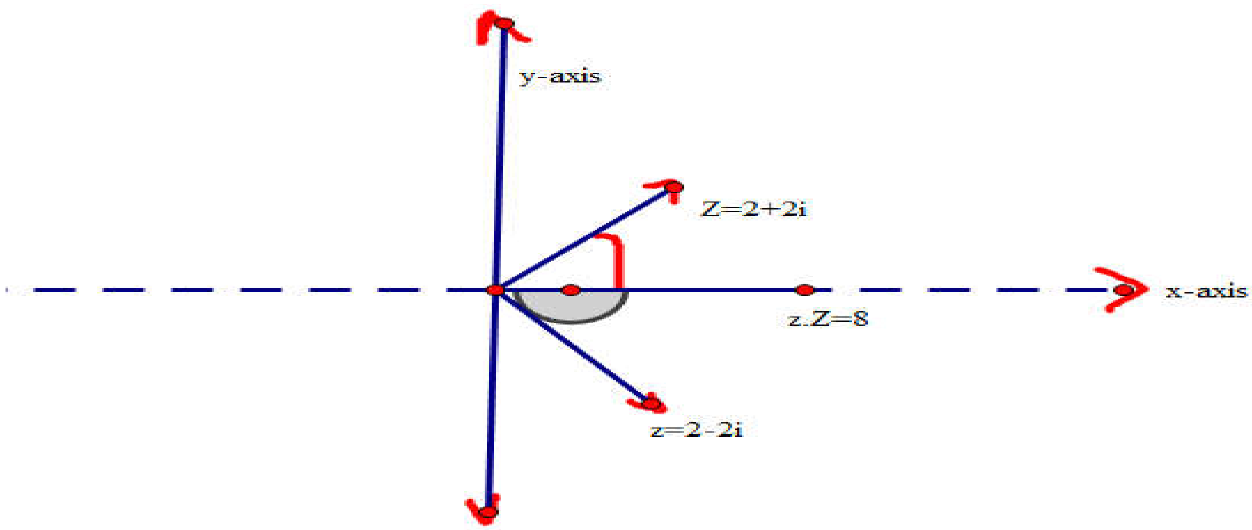
$$w = z_1 * z_2 + 2(3 - 3i) = 8 + 6 - 6i = 14 - 6i$$

   Here by the formula

   $C_k = r^{1/n} e^{\frac{i(\theta + 2\pi k)}{n}}$, where k=0,1,2,3 and $r = \sqrt{196 + 36} = \sqrt{232} \approx 15 \ and \ tan^{-1}\left(\frac{-6}{14}\right) \approx -23.2 = 336.7°$

   If $k = 0, we \ get \ C_0 = 15^{1/4} e^{\frac{336.7° i}{4}}$ continue like this for the values of $k = 1,2, and \ 3$

c. Represent the two complex number and its product in the same z-plane and find the angle between each complex number. For Natural science extreme only



Find the angle between

i. Z and z

Graphically it is obvious.

But one can also calculate the angle between each complex number by considering the complex as a vector.

Thus, to find the angle between $z_1 \,\&\, z_2$ as $z_1.z_2 = |z_1||z_2|\cos\theta$ by using dot product

$$\Rightarrow 4-4 = 8\cos\theta \Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}$$

ii. Z and 8

To this again we have $8.(2+2i) = 16 = 8.2\sqrt{2}\cos\vartheta \Rightarrow \vartheta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

iii. 8 and z

To this again we have $8.(2-2i) = 16 = 8.2\sqrt{2}\cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

7. Let $(x+iy)^{1/3} = s + it \ where\ x, y, s, t \in R$. Then compute $tx - sy$.

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   Solution

$$x + iy = s^3 + i^3t^3 + 3ist(s + it)$$

   $= s^3 - it^3 + 3s^2ti - 3st^2$ this is true if and only if $x = s^3 - 3st^2$ and $y = -t^3 + 3s^2t$

   Hence the required answer by using this yield to

$$tx - sy = -2st(s^2 + t^2)$$

8. If z and w are two complex numbers such that $|zw| = 1$and $arg\ (z) - arg\ (w)\ = \frac{\pi}{2}$.

   Then compute$\bar{z}w$ For Natural science extreme only

   Solution

   Let $z = r_1 e^{i\theta_1} and w = r_2 e^{i\theta_2}$ and $|zw| = \left|r_1 e^{i\theta_1} r_2 e^{i\theta_2}\right| = 1 \Rightarrow |r_1 r_2| = 1 \ \&\ \theta_1 - \theta_2 = \frac{\pi}{2}$ from some sequence of calculation $\bar{z}w = -i$

9. Write a short note on types of function and give one example for each type by graphs. For social science extreme only

10. Write the inverses of each types of functions and with its graphs.

11. Evaluate $2^{log_2^{\sqrt{5}}}$ For social science extreme only

12. If $g(x) = log_3^{(x^2-4x+3)}$. Find $g(4)$ and the domain of g. For social science extreme only

Solution

Trivially

$$g(4) = \log_3^{16-12+3} = \log_3^7 = \frac{\log 7}{\log 3} = 1.77$$